

On scattering of electromagnetic waves by a wormhole

A.A. Kirillov, E.P. Savelova

Dubna International University of Nature, Society and Man,
Universitetskaya Str. 19, Dubna, 141980, Russia

Abstract

We consider scattering of a plane electromagnetic wave by a wormhole. It is found that the scattered wave is depolarized and has a specific interference picture depending on parameters of the wormhole and the distance to the observer. It is proposed that such features can be important in the direct search of wormholes.

1. As it was shown recently all features of cold dark matter models (CDM) can be reproduced by the presence of a gas of wormholes [1]. At very large scales wormholes behave exactly like very heavy particles, while at smaller subgalactic scales wormholes strongly interact with baryons and cure the problem of cusps. Therefore, we may state that up to date wormholes give the best candidate for dark matter particles. The final choice between different dark matter candidates seem to require a direct observation of a cosmological wormhole. Possible observational effects of wormholes attract the more increasing attention e.g. see Refs. [2] and references therein. In this paper we consider the most simplest effect which may be used in observations. We demonstrate that the scattering of a plane electromagnetic wave by a wormhole leads to a partial depolarization and a specific interference picture in the scattered signal. The interference picture depends on the distance to the observer and specific parameters of a wormhole giving thus an instrument for the possible search of wormholes.

2. The simplest wormhole is described by the metric

$$ds^2 = c^2 dt^2 - h^2(r) \delta_{\alpha\beta} dx^\alpha dx^\beta, \quad (1)$$

where

$$h(r) = 1 + \theta(b - r) \left(\frac{b^2}{r^2} - 1 \right) \quad (2)$$

and $\theta(x)$ is the step function. Such a wormhole has vanishing throat length. Indeed, in the region $r > b$, $h = 1$ and the metric is flat, while the region $r < b$, with the obvious transformation $y^\alpha = \frac{b^2}{r^2} x^\alpha$, is also flat for $y > b$. Therefore, the regions $r > b$ and $r < b$ represent two Minkowski spaces glued at the surface of a sphere S^2 with the centre at the origin $r = 0$ and radius $r = b$. Such a

space can be described with the ordinary double-valued flat metric in the region $r_{\pm} > b$ by

$$ds^2 = c^2 dt_{\pm}^2 - \delta_{\alpha\beta} dx_{\pm}^{\alpha} dx_{\pm}^{\beta}, \quad (3)$$

where the coordinates x_{\pm}^{α} describe two different sheets of space.

A generalization appears when we change the step function $\theta(x)$ with any smooth function $\tilde{\theta}(x)$ which has the property $\tilde{\theta}(x) \rightarrow 0$ as $x \rightarrow -\infty$ and $\tilde{\theta}(x) \rightarrow 1$ as $x \rightarrow 0$. In confront to the previous case, such a wormhole will have a non-vanishing throat length. However, while it differs in details, the general features remain the same. Such wormholes have vanishing mass, but one may also include a non-vanishing wormhole mass as described by [3]. In particular, as it was shown in Ref. [1] the background density of baryons automatically generates a some finite rest mass value. Save the exotic matter which is necessary to support the cosmological wormhole, it (as a massive object) has also to be surrounded with standard forms of matter (dust, gas, etc.) which may complicate the direct observation of wormholes.

Identifying the inner and outer regions of the sphere S^2 allows the construction of a wormhole which connects regions in the same space (instead of two independent spaces). This is achieved by gluing the two spaces in (3) by motions of the Minkowski space (the Poincare motions). If $R_+^{\mu} = (t_+, \vec{R}_+)$, where \vec{R}_+ is the position of the sphere in coordinates x_+^{μ} , then the gluing is the rule

$$x_+^{\mu} = R_+^{\mu} + \Lambda_{\nu}^{\mu} (x_-^{\nu} - R_-^{\nu}), \quad (4)$$

where $\Lambda_{\nu}^{\mu} \in SO(3,1)$, which represents the composition of a translation and a Lorentz transformation of the Minkowski space. In terms of common coordinates such a wormhole represents the standard flat space in which the two cylinders $S_{\pm}^2 \times R^1$ (with centers at positions \mathbf{R}_{\pm}) are glued by the rule (4). Thus, in general, the wormhole is described by a set of parameters: the throat radius b , positions and velocities of throats \mathbf{R}_{\pm} , \mathbf{V}_{\pm} , spatial rotation matrix $U_{\beta}^{\alpha} \in O(3)$, and in general some additional time shift $\Delta t = t_+ - t_-$. For the sake of simplicity we, in the present paper, suppose that throats does not move in space $\mathbf{V}_{\pm} \approx 0$ and the time shift is absent $\Delta t \approx 0$. The motion of one throat (say S_+) can be excluded by the choice of the reference system, while the possible respective motion of the second throat is supposed to have a small non-relativistic velocity $\mathbf{V}_- \ll c$.

3. The problem of the scattering of radiation on a static gas of wormholes has been considered recently in Ref. [4]. In particular it was demonstrated that any discrete source turns out to be surrounded with a diffuse halo which should be correlated with analogous halo of dark matter. However, we used there an approximation in which the size of wormholes was negligible. Here we consider some general features of the scattering by a wormhole of a finite size and account for the vector character of radiation. We point out Ref. [5] where some aspects of the scattering by an Ellis geometry were also considered.

Scattering of electromagnetic waves in the short-wavelength limit by a conducting sphere is the classical problem of electrodynamics [6]. We do not repeat

all derivations here referring to the above textbook but explain additional features which come from a wormhole.

Let $\mathbf{E} = \mathbf{E}_0 e^{-i\omega t + i\mathbf{k}_0 \mathbf{r}}$ be incident field, then the surface of a sphere can be divided into illuminated and shadow regions which produce contributions to the scattered field as

$$\mathbf{E}_{sh} \approx ikb^2 \frac{J_1(kb \sin \theta)}{kb \sin \theta} \frac{e^{-i\omega t + ikr}}{r} \frac{[[\mathbf{k}\mathbf{E}_0]\mathbf{k}]}{k^2} \quad (5)$$

from the shadow region and

$$\mathbf{E}_{ill} \approx -\frac{b}{2} E_0 \frac{e^{-i\omega t + ikr}}{r} e^{-2ikb \sin \frac{\theta}{2}} \mathbf{e}_0 \quad (6)$$

from the illuminated region respectively. Here b is the radius of the sphere,

$$\mathbf{e}_0 = (2(\mathbf{n}_0 \mathbf{E}_0) \mathbf{n}_0 - \mathbf{E}_0) / E_0,$$

\mathbf{n}_0 is the unit vector along $\mathbf{k} - \mathbf{k}_0$, $\mathbf{k} = k\mathbf{r}/r$ and $\cos \theta = (\mathbf{k}\mathbf{k}_0)/k^2$. It is also supposed that the sphere is at the origin. The contribution from the shadow region represents the standard diffraction which does not depend on the nature of the obstacle (it depends on the projected area only) and gives a very narrow beam along the incident signal $\theta \lesssim 10/kb$, while the illuminated region gives an isotropic intensity of radiation.

Now we add features from the structure of a wormhole. It turns out that a general wormhole can be considered as a couple of dielectric spheres S_+ and S_- glued along the surface. Thus the incident signal generates illuminated and shadow regions on the outer side of surface and analogous illuminated and shadow regions on the inner side of the surface. Due to the gluing the inner side of S_+ corresponds to the outer side of S_- and vice versa. Since points on the spheres are glued both throats radiate the scattered signal in the same manner and we may consider radiation from one throat. The incident field is partially reflected by the throat (in the illuminated region) and partially goes through the throat. The reflection and transmittance coefficients $|r|^2 + |t|^2 = 1$ depend on the specific structure of the wormhole and, in particular, on the matter content filling the throat and surrounding the wormhole (E.g., in the vacuum case $r(b, k) \rightarrow 0$ as $bk \gg 1$, e.g., see Refs. [5]). Thus the total scattered signal comprises an additional term as follows

$$\mathbf{E} = \mathbf{E}_{sh} + r\mathbf{E}_{ill} + t'\mathbf{E}'_{ill}. \quad (7)$$

Let S_+ be the throat at the origin. The additional term has the same form as (6) and describes the wave transmitted through the throat (i.e., radiation of the wave adsorbed on the conjugated throat S_-). Such term is equivalent to the classical reflection by the sphere S_+ of an additional wave $\mathbf{E}' = \mathbf{E}'_0 e^{-i\omega' t + i\mathbf{k}'_0 \mathbf{r} + i\psi}$ where $\psi = -\omega \Delta t + \mathbf{k}_0 \Delta \mathbf{R}$ ($\Delta \mathbf{R} = \mathbf{R}_- - \mathbf{R}_+$) is the phase difference which the incident field has in the center of the throat S_- , and \mathbf{E}'_0 , ω' , and \mathbf{k}'_0 are related to \mathbf{E}_0 , ω , and \mathbf{k}_0 by the Lorentz transformation Λ^μ_ν which defines the gluing (4).

The shadow contribution \mathbf{E}_{sh} gives a very narrow beam along \mathbf{k}_0 , while the illuminated parts define an omnidirectional flux $\mathbf{E}_{ill}^{tot} = r\mathbf{E}_{ill} + t'\mathbf{E}'_{ill}$ which is depolarized (since in general $\mathbf{e}_0 \neq \mathbf{e}'_0$). In the case when the wormhole is surrounded with a dense plasma $t' \rightarrow 0$ (as $bk \gg 1$) and the scattered signal does not differ from the reflection by a conducting sphere [6], while in the vacuum case $r \rightarrow 0$ (as $bk \gg 1$) [5] and the scattered signal corresponds to the reflection by a conducting sphere of the auxiliary wave \mathbf{E}' which relates to the incident wave by the Lorentz transformation (4). In the last case the scattered signal has, in general, a somewhat different wavelength k' and a different polarization \mathbf{e}'_0 as compared to the standard conducting sphere. In other words, we may state that in comparison to the scattering by a conducting sphere the wormhole leads to some additional shift of the wavelength and a rotation of the polarization. In the most general intermediate case the scattered signal shows a more complex interference picture. The intensity of the flux $I = (\mathbf{E}\mathbf{E}^*)$ is given by

$$I = \frac{b^2}{4} \frac{E_0^2}{r^2} \left(1 + A \cos \left\{ \psi - \Delta k (ct - r) - \left(2kb \sin \frac{\theta}{2} - 2k'b \sin \frac{\theta'}{2} \right) \right\} \right). \quad (8)$$

Here $\cos \theta = (\mathbf{r}\mathbf{k}_0) / rk_0$, $\cos \theta' = (\mathbf{r}\mathbf{k}'_0) / rk_0$, $\Delta k = c(\omega - \omega')$, and $A = (rt'^* + r^*t')(\mathbf{e}_0\mathbf{e}'_0)$

We see that the most strong interference picture appears when $A \simeq 1$ (recall that A depends on the specific structure and the matter composition of the wormhole) and is given by the phase $\Delta k (ct - r)$ which comes from the possible respective motions of throats $\Delta\omega \simeq k\Delta V$ (here $\Delta\mathbf{V} = \mathbf{V}_+ - \mathbf{V}_-$ and c is the speed of light) and $\Delta\omega$ is the standard Doppler shift. In this case the phase ψ has also a dependence on time (via $\mathbf{R}_\pm(t)$). However the additional phase in (8) forms a more peculiar and complex interference picture with a much larger wavelength $\delta k \ll \Delta k$ which in principle can be measured while the Earth orbits the Sun. We point out that these oscillations are not pure harmonic ones.

Consider for simplicity the case when the Lorentz transformation reduces to a pure spatial rotation U_β^α and the frequency remains the same $\omega' = \omega$. Then expanding (8) by $\delta\mathbf{r}$ near the observer at the position \mathbf{r} we find the expression

$$I = \frac{b^2}{4} \frac{E_0^2}{r^2} (1 + A \cos \{ \varphi_0 + (\delta\mathbf{k}\delta\mathbf{r}) \})$$

where the constant phase $\varphi_0 = \psi - 2kb \left(\sin \frac{\theta}{2} - \sin \frac{\theta'}{2} \right)$, and

$$\delta\mathbf{k} = \left[\frac{1}{\sin \frac{\theta}{2}} - \frac{1}{\sin \frac{\theta'}{2}} \right] \frac{2b}{r} \mathbf{k}_0. \quad (9)$$

Thus the intensity has a specific additional oscillations with a very big wavelength proportional to the ratio $\lambda_0 r / b \gg \lambda_0$ where λ_0 is the wavelength of the incident signal. We suppose that the interference picture described above should be taken into account in analyzing observations and may be useful in the search of wormholes.

This research was supported in some part by RFBR 09-02-00237-a and by a Royal Society grant

References

- [1] Kirillov, A.A., Savelova E.P., 2011, MNRAS, arXiv:1006.3230; 2008 Phys. Lett. **B660**, p. 93-99.
- [2] F. Abe 2010 ApJ **725**, 787, Pozanenko, A., Shatskiy A., To the search for observational evidence of wormholes, eprint arXiv:1007.3620, Cramer, J.G., et al., 1995 Phys.Rev. **D 51**, 3117, Nandi, K.K., Zhang, Y.-Z., Zakharov, A.V., 2006 Phys.Rev. **D 74**, 024020, Shatskii, A. A., 2004 Astron. Rep. **48**, 525; Kardashev N.S., Novikov I.D., Shatskiy A.A., 2007 Int.J.Mod.Phys. **D16** 909-926.
- [3] Visser M., 1996, Lorentzian wormholes, Springer-Verlag, New-York, Inc.
- [4] Kirillov A.A., Savelova E.P., Shamshutdinova G.D., 2009, JETP Lett., 90, 599-603
- [5] Clement, G. 1984 Int. Journ. Theor. Phys., **23**, pp.335-350; Perez Bergliaffa S.E., Hibberd K.E. 2000 Phys. Rev. **D62** 044045.
- [6] Jackson J.D., Classical Electrodynamics, ed. (Wiley, New York, 1962).